Name: **Version #1**

Instructor and class time: ______________________

Math 10550, Final
Apr. 29, 2026

- The Honor Code is in effect for this examination. All work is to be your own.
- Please turn off all cellphones and electronic devices.
- Calculators are **not** allowed.
- The exam lasts for 2 hours.
- Be sure that your name and instructor's name are on the front page of your exam.
- Be sure that you have all 16 pages of the test.
- There are 26 questions, each question is worth 5 points.
- Your score will be the sum of the best 24 scores.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(•)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(•)
			2		
3.	(a)	(b)	(•)	(d)	(e)
4.	(a)	(b)	(c)	(•)	(e)
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5.	(a)	(•)	(c)	(d)	(e)
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13.	(a)	(b)	(c)	(•)	(e)
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16.	(a)	(b)	(c)	(d)	(•)
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17.	(a)	(b)	(•)	(d)	(e)
18.	(a)	(•)	(c)	(d)	(e)
			10		
19.	(a)	(b)	(•)	(d)	(e)
20.	(•)	(b)	(c)	(d)	(e)
			11		
21.	(a)	(b)	(c)	(•)	(e)
22.	(a)	(b)	(c)	(•)	(e)
			12		
23.	(•)	(b)	(c)	(d)	(e)
24.	(a)	(b)	(c)	(d)	(•)
			13		
25.	(a)	(b)	(•)	(d)	(e)
26.	(a)	(b)	(c)	(•)	(e)
			14		

Please do NOT write in this box.

Total ______________________

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2. (a) (b) (c) (d) (e)

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20. (a) (b) (c) (d) (e)

.... 11 ....

21. (a) (b) (c) (d) (e)
22. (a) (b) (c) (d) (e)

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23. (a) (b) (c) (d) (e)
24. (a) (b) (c) (d) (e)

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25. (a) (b) (c) (d) (e)
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Total ______________________________

1.(5pts) If $f(x) = \int_0^{5x} \cos(u^2)du$, find $f'(x)$

Solution. We want to apply the Fundamental Theorem of Calculus, but cannot immediately. So we define

$$g(x) = \int_0^{x} \cos(u^2)du$$

to get $f(x) = g(5x)$. By the chain rule, $f'(x) = 5g'(x)$. Now we can apply the Fundamental Theorem to g:

$$f'(x) = 5g'(x) = 5\cos((5x)^2) = 5\cos(25x^2).$$

(a) $5\cos(25x^2)$ (b) $-5\cos(25x^2)$ (c) $-25\cos(5x^2)$ (d) $5\cos(5x^2)$ (e) $-\cos(5x^2)$

2.(5pts) Find all local maxima and minima of the function $f(x) = 2|x| - x^2 - 1$.

Solution. Taking the derivative of $2x - x^2 - 1$ for $x > 0$ gives $f' = 2 - 2x$ which means a critical point is at $x = 1$. Taking the derivative again and using the second derivative test gives $x = 1$ is a local maxima.

Taking the derivative of $-2x - x^2 - 1$ for $x < 0$ gives $f' = -2 - 2x$ which means a critical point is at $x = -1$. Taking the derivative again and using the second derivative test gives $x = -1$ is a local maxima.

When $x < 0$ the function is increasing and when $x > 0$ the function increases for small values away from 0 so it is easy to see that $x = 0$ is a local minima.

(a) Only local minimum at $(x, y) = (0, -1)$, no local maxima.

(b) Local maximum: $(x, y) = (-1, 0)$, local minimum $(x, y) = (0, -1)$.

(c) Local maxima: $(x, y) = (-1, 0)$ and $(x, y) = (1, 0)$, no local minimum.

(d) No local maxima or minima, because the function $|x|$ has no derivative at $x = 0$.

(e) Local maxima: $(x, y) = (-1, 0)$ and $(x, y) = (1, 0)$, local minimum $(x, y) = (0, -1)$.

3.(5pts) Find $f'(x)$ for

$$f(x) = \ln(2^x + x) + \arcsin(e^x)$$

Solution:

$$f'(x) = \frac{d(2^x + x)/dx}{2^x + x} + \frac{d(e^x)/dx}{\sqrt{1-(e^x)^2}}.$$

$$= \frac{2^x \ln 2 + 1}{2^x + x} + \frac{e^x}{\sqrt{1-e^{2x}}}$$

(a) $\dfrac{e^x \ln 2 + 1}{2^x + x} + \dfrac{e^x}{\sqrt{e^{2x}-1}}$

(b) $\dfrac{2^x + 1}{2^x + x} + \dfrac{1}{\sqrt{1-e^{2x}}}$

(c) $\dfrac{2^x \ln 2 + 1}{2^x + x} + \dfrac{e^x}{\sqrt{1-e^{2x}}}$

(d) $\dfrac{2^x \ln 2}{2^x + x} + \dfrac{e^x}{1+e^{2x}}$

(e) $\dfrac{e^x \ln 2}{2^x + x} + \dfrac{e^x}{\sqrt{1-2e^x}}$

4.(5pts) Compute $\displaystyle\lim_{x\to 0^+} \frac{x^2-9}{\sin x}$.

Solution

If x is close to 0 but larger than 0, then the denominator $\sin x$ is a small positive number and $x^2 - 9$ is close to -9. So the quotient $\dfrac{x^2-9}{\sin x}$ is a *large negative* number. So

$$\lim_{x\to 0^+} \frac{x^2-9}{\sin x} = -\infty.$$

(a) $+\infty$

(b) -9

(c) 0

(d) $-\infty$

(e) Does not exist and is not $+\infty$ or $-\infty$.

5.(5pts) Let $F(x) = f(g(x))$. Compute $F'(2)$ using the following information:

$f(-1) = -3,\ f(2) = 12,\ g(-1) = -7,\ g(2) = -1,$

$f'(-1) = 2,\ f'(2) = 8,\ g'(-1) = -1,\ g'(2) = 5.,$

Solution. Using the chain rule, $F'(x) = f'(g(x))g'(x)$, so $F'(2) = f'(g(2))g'(2) = f'(-1) \cdot 5 = 2 \cdot 5 = 10$

(a) 2 (b) 10 (c) 52 (d) -15 (e) 40

6.(5pts) Evaluate $\lim_{x\to+\infty}(\sqrt{x^2 - x} - \sqrt{x^2 + 5x}\ )$.

Solution

$$\begin{aligned}
\lim_{x\to\infty}(\sqrt{x^2 - x} - \sqrt{x^2 + 5x}) &= \lim_{x\to\infty}(\sqrt{x^2 - x} - \sqrt{x^2 + 5x})\frac{\sqrt{x^2 - x} + \sqrt{x^2 + 5x}}{\sqrt{x^2 - x} + \sqrt{x^2 + 5x}} \\
&= \lim_{x\to\infty}\frac{(x^2 - x) - (x^2 + 5x)}{\sqrt{x^2 - x} + \sqrt{x^2 + 5x}} \\
&= \lim_{x\to\infty}\frac{-6x}{\sqrt{x^2(1 - \frac{1}{x})} + \sqrt{x^2(1 + \frac{5}{x})}} \\
&= \lim_{x\to\infty}\frac{-6x}{x\sqrt{1 - \frac{1}{x}} + x\sqrt{1 + \frac{5}{x}}} \\
&= \lim_{x\to\infty}\frac{-6}{\sqrt{1 - \frac{1}{x}} + \sqrt{1 + \frac{5}{x}}} \\
&= \frac{-6}{\sqrt{1 - 0} + \sqrt{1 + 0}} = -3.
\end{aligned}$$

(a) Does not exist

(b) 0

(c) -3

(d) -6

(e) 3

7.(5pts) A page of a book is to have a total area of 150 square inches, with 1 inch margins at the top and sides, and a 2 inch margin at the bottom. Find the dimensions in inches of the page which will have the largest print area.

Solution. You should draw a picture for this problem. If l is the total length of the page, w is the total width, and A is the print area, then

$$lw = 150, A = (l-2)(w-3).$$

We want to maximize A so we want to substitute in for one of the variables so that we can take the derivative. Note $l = 150/w$. Therefore,

$$A(w) = (\frac{150}{w} - 2)(w-3) = 150 - \frac{450}{w} - 2w + 6,$$

and

$$A'(w) = \frac{450}{w^2} - 2.$$

If $A'(w) = 0$ then $w^2 = 225$ so $w = 15$. The first derivative test easily shows this gives a maximum area. Since $l = 150/w$, $l = 10$.

(a) $5\sqrt{3} \times \frac{30}{\sqrt{3}}$ (b) 10×15 (c) $3\sqrt{7} \times \frac{50}{\sqrt{7}}$ (d) $11\frac{7}{13} \times 13$ (e) 5×30

8.(5pts) Compute the tangent line to the ellipse given by the equation $x^2 + 4y^2 = 5$ at the point $(1, -1)$

Solution. If we take the derivative with respect to both sides of the equation we see $2x + 8y\frac{dy}{dx} = 0$, or $\frac{dy}{dx} = -\frac{x}{4y}$. So the slope at $(1, -1)$ is $\frac{1}{4}$. Thus the tangent line at $(1, -1)$ is

$$y + 1 = \frac{1}{4}(x-1),$$

or

$$y = \frac{1}{4}x - \frac{5}{4}.$$

(a) $y = \frac{1}{4}x - \frac{5}{4}$

(b) $y = -\frac{1}{4}x - \frac{3}{4}$

(c) The tangent line does not exist.

(d) $y = \frac{1}{4}x - \frac{3}{4}$

(e) $y = \frac{1}{2}x - \frac{3}{2}$

9.(5pts) A kite 100 ft above the ground is flying horizontally (away from its holder) with a speed of 16ft/sec. At what rate is the angle between the string and the horizontal direction changing, when 200 ft of the string have been let out?

Solution. The kite is at a constant height of 100ft with a length of x ft away. So using some trigonometry, we see that if θ is the angle between the string and the horizontal direction, $\tan\theta = 100/x$. Taking the derivative with respect to t gives $\sec^2\theta \frac{d\theta}{dt} = -100x^{-2}\frac{dx}{dt} = -1600x^{-2}$ since the kite is flying away at 16ft/sec. When 200ft have been let out, $\sin\theta = 1/2$ and $\theta = \pi/6$. At this value of θ we have $4/3\frac{d\theta}{dt} = -\frac{1600}{200^2 - 100^2}$ or

$$\frac{d\theta}{dt} = -\frac{1200}{30000} = -\frac{1}{25}\frac{radians}{second}.$$

(a) $-\frac{1}{25}$ radian/second

(b) $\frac{1}{25}$ radian/second

(c) $\frac{1}{50}$ radian/second

(d) $-\frac{1}{50}$ radian/second

(e) $\frac{\pi}{50}$ radian/second

10.(5pts) Let $f(x) = \begin{cases} ax+1 & x < 0, \\ x^2+1 & x \geq 0. \end{cases}$

For what constant a is f differentiable everywhere?

Solution

f is clearly differentiable for $x < 0$ and for $x > 0$. For $x < 0$, $f'(x) = a$, so $\lim_{x\to 0^-} f'(x) = a$. For $x > 0$, $f'(x) = 2x$, so $\lim_{x\to 0^+} f'(x) = 0$. For f to be differentiable at 0, we need $a = \lim_{x\to 0^-} f'(x) = \lim_{x\to 0^+} f'(x) = 0$.

(a) No value of a

(b) $a = 1$

(c) $a = 0$

(d) $a = 2$

7. Initials: ____________

(e) Any value of a

11.(5pts) Compute $\lim_{x\to 2^-} \frac{x^2-4}{x^2-5x+6}$.

Solution

When $x \neq 2$,

$$\frac{x^2-4}{x^2-5x+6} = \frac{(x-2)(x+2)}{(x-2)(x-3)} = \frac{x+2}{x-3}.$$

So

$$\lim_{x\to 2^-} \frac{x^2-4}{x^2-5x+6} = \frac{2+2}{2-3} = -4.$$

(a) -4 (b) $-\infty$ (c) 1 (d) $+\infty$ (e) 0

12.(5pts) For $y = (\sin 4x)^8$, compute y'.

Solution. Using the chain rule a total of three times, we get

$$\begin{aligned} y' =& 8 \cdot (\sin 4x)^7 \cdot \frac{d}{dx}(\sin(4x)) = 8 \cdot (\sin 4x)^7 \cdot \cos 4x \cdot \frac{d}{dx}4x \\ =& 32(\sin 4x)^7 \cos 4x. \end{aligned}$$

(a) $32(\cos 4x)^7$

(b) $32(\sin 4x)^7$

(c) $32(\sin 4x)^7 \cos 4x$

(d) $8(\cos 4x)^7$

(e) $8(\sin 4x)^7$

13.(5pts) Evaluate the integral $\int_0^{\sqrt{\pi}} x\sin(x^2)dx$.

Solution. Let $u = x^2$. Then $du = 2xdx$ and

$$\int_0^{\sqrt{\pi}} x\sin(x^2)dx = \frac{1}{2}\int_0^{\pi} \sin(u)du = -\frac{1}{2}\cos(u)|_0^{\pi} = 1/2 + 1/2 = 1.$$

(a) $\frac{\pi}{4}$ (b) 2 (c) $1 - \frac{1}{\pi}$ (d) 1 (e) $\frac{1}{4}$

14.(5pts) Which of the following integrals give the area of the region below the curve $y = 2x$ and above the curve $y = x^2 - 4x$?

Solution. You should draw a picture. We find the intersection points of the two curves by solving

$$2x = x^2 - 4x \Leftrightarrow x^2 - 6x = 0 \Leftrightarrow x(x-6) = 0 \Leftrightarrow x = 0, 6.$$

For small x, $x^2 - 4x < 0$. In $[0, 6]$, the curve $y = x^2 - 4x$ is below $y = 2x$. This can be seen by taking, say, $x = 1$. Therefore the area between the curves is given by

$$\int_0^6 \Big(2x - (x^2 - 4x)\Big)dx.$$

(a) $\int_0^4 \Big((x^2 - 4x) - 2x\Big)dx$

(b) $\int_0^6 \Big(2x - (x^2 - 4x)\Big)dx$

(c) $\int_0^4 \Big(2x - (x^2 - 4x)\Big)dx$

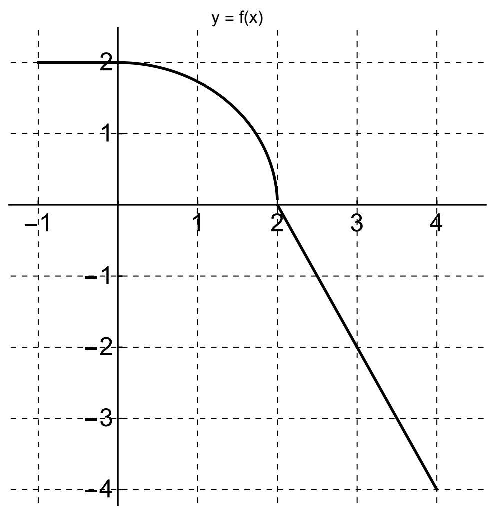
(d) $\int_0^6 \Big((x^2 - 4x) - 2x\Big)dx$

(e) $\int_0^4 \Big(2x - (x^2 - 4x)\Big)dx + \int_4^6 \Big((x^2 - 4x) - 2x\Big)dx$

15.(5pts) The graph shown below is that of $f(x)$ for $-1 \leq x \leq 4$ where

$$f(x) = \begin{cases} 2 & \text{if} -1 \leq x \leq 0 \\ \sqrt{4-x^2} & \text{if} 0 < x \leq 2 \\ 4-2x & \text{if} 2 \leq x \leq 4 \end{cases}$$

Which of the following equals $\int_{-1}^{4} f(x)dx$?



Solution: From -1 to 0, the function is constant with output 2. So the area under the curve is given by the area of a rectangle with base 1 and height 2. The area of such a rectangle is 2. From 0 to 2, we see that the function traces out the upper left portion of a circle with radius 2 centered at $(0,0)$ and hence, we have that the area is given by $\frac{1}{4}\pi 2^2 = \pi$. From 2 to 4, we see that the curve traces the hypotenuse of a right triangle with base 2 and height 4. The area of such a triangle is $\frac{1}{2}(2)(4) = 4$. However, since function is under the $x-$axis from 2 to 4, we need to account for this by subtracting the area of the triangle. So the integral is evaluated to be $2 + \pi - 4 = \pi - 2$.

(a) $2\pi - 2$ (b) $6 + \pi$ (c) $\pi - 2$ (d) π (e) 0

16.(5pts) Find the **left endpoint approximation** to the definite integral

$$\int_{-1}^{3} \frac{6}{2+x}dx$$

using four approximating rectangles of equal base width.

Solution: The left endpoint approximation uses the left endpoint of a subinterval:
$\int_a^b f(x)dx \approx \Delta x\left(f(x_0) + f(x_2) + \cdots + f(x_{n-1})\right)$ where $\Delta x = \dfrac{b-a}{n}$.

We have that $f(x) = \dfrac{6}{2+x}$, $a = -1$, $b = 3$, $n = 4$, Therefore, $\Delta x = \dfrac{3-(-1)}{4} = 1$.

Divide the interval into 4 subintervals of the length $\Delta x = 1$ with the following endpoints: $-1, 0, 1, 2, 3$.

Now, just evaluate the function at the left endpoints of the subintervals.

$f(-1) = 6$, $f(0) = 3$, $f(1) = 2$, $f(2) = \frac{3}{2}$.

Finally, just sum up the above values and multiply Δx we obtain that

$$\int_{-1}^{3} \frac{6}{2+x}dx \approx 1 * \left(6 + 3 + 2 + \frac{3}{2}\right) = \frac{25}{2}.$$

(a) $\dfrac{71}{5}$ (b) $\dfrac{71}{10}$ (c) 25 (d) $\dfrac{131}{10}$ (e) $\dfrac{25}{2}$

17.(5pts) Compute the derivative y' for the curve $\sqrt{x^2+y^2} = 2+y$ at the point $x=4$, $y=3$

Solution. Taking the derivative of both sides of the equation and using the chain rule gives $1/2(x^2+y^2)^{-1/2}(2x+2yy') = y'$. So evaluating at the point $x=4, y=3$, we get $1/2(4^2+3^2)^{-1/2}(2\cdot 4+2\cdot 3y') = y'$ gives $1/10(8+6y') = y'$ which we can solve for to get $y'=2$.

(a) -2 (b) 0 (c) 2 (d) $2/11$ (e) $-2/11$

18.(5pts) Find the derivative of $(x^2+1)^{x^2+1}$.

Solution: We use logarithmic differentiation. Let $y=(x^2+1)^{x^2+1}$. Then

$$\ln y = (x^2+1)\ln(x^2+1).$$

Differentiating both sides with respect to x, we get

$$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}(x^2+1)\ln(x^2+1) = 2x\ln(x^2+1) + \frac{2x(x^2+1)}{(x^2+1)} = 2x\big[\ln(x^2+1)+1\big].$$

Multiplying both sides by y, we get

$$\frac{dy}{dx} = y2x\big[\ln(x^2+1)+1\big] = (x^2+1)^{x^2+1}2x\big[\ln(x^2+1)+1\big]$$

(a) This function is not defined and hence has no derivative.

(b) $(x^2+1)^{x^2+1}\,2x\big(\ln(x^2+1)+1\big)$

(c) $2x(x^2+1)^{x^2}$

(d) $(x^2+1)^{x^2+1}$

(e) $(x^2+1)^{x^2+1}\big(2x\ln(x^2+1)\big)$

19.(5pts) Which of the following expressions is equal to $\int_0^2 \sin(x^2)dx$?

Solution: For $f(x) = \sin(x^2)$, we know that

$$\int_0^2 \sin(x^2)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

where for the interval $[0, 2]$, $x_i = \frac{2i}{n}$ and $\Delta x = \frac{2}{n}$. Thus,

$$\int_0^2 \sin(x^2)dx = \lim_{n\to\infty} \sum_{i=1}^{n} \sin\left(\frac{4i^2}{n^2}\right)\frac{2}{n}$$

(a) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n} \sin\left(\frac{4i^2}{n^2}\right)$

(b) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n} \sin\left(\frac{i^2}{n^2}\right)$

(c) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n} \sin\left(\frac{4i^2}{n^2}\right)$

(d) $-\cos(4) + 1$

(e) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n} \sin\left(\frac{2i}{n}\right)$

20.(5pts) If $f(x)$ is a continuous function with

$$\int_{-2}^{-1} f(x)\,dx = 2, \quad \int_{-2}^{2} f(x)\,dx = 1 \text{ and } \int_{2}^{5} f(x)\,dx = 2$$

find $\int_{-1}^{5} f(x)\,dx$.

Soution: $\int_{-1}^{5} f(x)dx = \int_{-2}^{2} f(x)dx + \int_{2}^{5} f(x)dx - \int_{-2}^{-1} f(x)dx = 1 + 2 - 2 = 1.$

(a) 1 (b) 3 (c) 2 (d) 0 (e) 6

21.(5pts) Compute $\lim_{x\to-\infty} \dfrac{\sqrt{4x^2+x+1}}{3x-1}$.

Solution.

Note $x = -\sqrt{x^2}$ for $x < 0$. Multiply the top and bottom of $\frac{\sqrt{4x^2+x+1}}{3x-1}$ by $\frac{1}{x}$ to get

$$\frac{\sqrt{4x^2+x+1}}{3x-1} = \frac{\frac{1}{x}\cdot\sqrt{4x^2+x+1}}{\frac{1}{x}\cdot(3x-1)} = -\frac{\sqrt{4+(1/x)+(1/x^2)}}{3-(1/x)}.$$

Hence the limit as $x \to -\infty$ is $-\dfrac{\sqrt{4}}{3} = -\dfrac{2}{3}$.

(a) $-1/3$ (b) $2/3$ (c) 0 (d) $-2/3$ (e) $1/3$

22.(5pts) Evaluate: $\int e^{\cos^2(x)}\sin(x)\cos(x)dx$

$$\begin{aligned}
\text{Let } u &= \cos^2(x) \\
du &= -2\cos(x)\sin(x)dx \\
\Rightarrow \frac{du}{-2} &= \cos(x)\sin(x)dx
\end{aligned}$$

$$\begin{aligned}
\Rightarrow \int e^{\cos^2(x)}\sin(x)\cos(x)dx &= \int e^u \frac{du}{-2} \\
&= -\frac{1}{2}\int e^u du = -\frac{1}{2}e^u + C = -\frac{1}{2}e^{\cos^2(x)} + C
\end{aligned}$$

(a) $-e^{\cos^2(x)} + C$

(b) $-\frac{1}{2}e^{\cos^2(x)}cos(x) + C$

(c) $e^{\cos^2(x)} + C$

(d) $-\frac{1}{2}e^{\cos^2(x)} + C$

(e) $-e^{\cos^2(x)}\sin(x)\cos(x) + C$

23.(5pts) Compute $\lim_{x\to 0} \dfrac{\tan 2x}{\sin 3x}$.

Solution

Using L'Hospital's rule, we get

$$\begin{aligned} \lim_{x\to 0} \frac{\tan 2x}{\sin 3x} &= \lim_{x\to 0} \frac{2\sec^2 2x}{3\cos 3x} \\ &= \frac{2}{3} \text{ by evaluation.} \end{aligned}$$

(a) 2/3 (b) 0 (c) 1/3 (d) 1 (e) 2

24.(5pts) How many inflection points does the curve $y = \dfrac{x^5}{5} + \dfrac{x^4}{4}$ have?

Solution. First we note $y' = x^4 + x^3$ and $y'' = 4x^3 + 3x^2 = x^2(4x+3)$. Hence $y'' = 0$ at $x = 0$ and $x = -3/4$. However, $y'' < 0$ in $(-\infty, -3/4)$ and $y'' > 0$ in $(-3/4, 0)$ and $(0, \infty)$. Hence only $-3/4$ is the inflection point.

(a) 4 (b) 0 (c) 3 (d) 2 (e) 1

25.(5pts) Find the linearization of $f(x) = \sqrt{10 - x^2}$ at $a = -1$.

Solution. The linearization of $f(x)$ at $a = -1$ is $L(x) = f(a) + f'(a)(x - a)$. At $a = -1$, $f(a) = 3$ and $f''(x) = 1/2(10 - x^2)^{-1/2} \cdot (-2x)$ so $f'(a) = 1/3$ and substituting back in gives $f(x) = 3 + 1/3(x + 1)$.

(a) $L(x) = -\frac{2}{3}(x + 1) + 3$

(b) $L(x) = -\frac{1}{3}(x + 1) + 3$

(c) $L(x) = \frac{1}{3}(x + 1) + 3$

(d) $L(x) = x + 4$

(e) $L(x) = \frac{2}{3}(x + 1) + 3$

26.(5pts) A particle is moving in a straight line with velocity $v(t) = \dfrac{4}{3}t^3 - \dfrac{4}{3}t$ feet per second. Find the distance travelled by the particle on the time interval $0 \leq t \leq 2$.

Solution: $v(t) = \dfrac{4}{3}t(t^2 - 1) = 0$ implies $t = \pm 1$ or $t = 0$. Hence, $t = 0$ and $t = 1$ are the only values for t in $[0, \infty)$ where $v(t) = 0$.

Observe that $v(t) < 0$ for $0 \leq t \leq 1$ and $v(t) > 0$ for $1 \leq v(t) \leq 2$. Total distance traveled is then:

$$\begin{aligned} \int_0^2 |v(t)|\, dt &= \int_0^1 -v(t)\, dt + \int_1^2 v(t)\, dt \\ &= \left[-\frac{1}{3}t^4 + \frac{2}{3}t^2\right]_0^1 + \left[\frac{1}{3}t^4 - \frac{2}{3}t^2\right]_1^2 \\ &= \left[-\frac{1}{3} + \frac{2}{3}\right] + \left[\frac{16}{3} - \frac{8}{3} - \frac{1}{3} + \frac{2}{3}\right] = \frac{10}{3} \end{aligned}$$

(a) $\dfrac{20}{3}$ feet

(b) $\dfrac{32}{3}$ feet

(c) 3 feet

(d) $\dfrac{10}{3}$ feet

(e) $\dfrac{8}{3}$ feet